Comparative Advantage and Spatial Interactions: Implications for GIS-Based Land Allocation Analysis

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Abstract:

Researchers have used GIS to solve land allocation problems using what is basically a comparative advantage process (CAP). Suitability maps are constructed showing the "production potential" of a variety of alternative "products" that could be produced from each region in a map. Thus, a suitability map describing the grazing potential of an area might show that a region with good soils and adequate rainfall can support a great deal of grazing, while an area with poor soils and/or little rainfall has a limited ability to support grazing. The basic CAP approach requires the researcher to develop one such suitability map for each possible land use under consideration. Once these suitability maps are created, each map is multiplied by the "value" of the product it represents (e.g., the grazing map would be multiplied by the unit value of grazing, a timber map would be multiplied by the unit value of timber, etc.). This produces a series of production value maps. The values in these value maps are compared to one another, and each region is assigned to the land use that maximizes the value of its products.

There are many difficulties in implementing this approach, including the fact that many of the "products" and "values" involved in this process concern nonmarket goods (e.g., open space), which makes production of suitability maps and derivation of unit values very difficult. The simple CAP-based process just described also ignores spatial interactions between regions. Thus, while it is logical to assume that a timber harvest area adjacent to a wildlife habitat zone might impact the wildlife zone, the simple CAP-based process just described ignores any such interactions. In this study, a modified version of the CAP-based land allocation process is presented. It will be demonstrated that this modified approach, which relies on goal programming techniques, can address the issue of spatial interactions between land uses.

INTRODUCTION

One of the most basic issues facing land managers is the land allocation problem. Simply put, this problem involves allocating each acre of land in some planning area to one or more potential uses. In reality, this problem involves a temporal component because land uses frequently change over time. Thus, a full reckoning of the land allocation problem would decide what use or uses each acre of land should be allocated to, for each year (or some other time period) of the planning horizon. However, this temporal dimension is frequently handled by defining the term "land use" to mean a sequence of activities over time (e.g., a single "land use" might involve assigning a piece of land to agricultural uses for years 1 through 10 and residential development for years 11 and later). Under this approach, the temporal component of the land allocation problem is handled in the land uses themselves, and the problem returns to one of assigning land uses to parcels of land. It is this temporally-explicit definition of "land use" that will be used throughout this study.

Land allocation is clearly a spatial issue, and thus it is tempting to look to geomatics for a solution to the land allocation problem. However, it is also true that deciding how resources (such as land) should be allocated among competing uses is a classic economics problem. Thus, an econometrics solution to the land allocation problem seems possible. Merging these two possible solution strategies -- geomatics and econometrics -- is an obvious and attractive way to approach the land allocation problem.

Researchers following this combined approach have developed a variety of geomatics and/or econometrics solution strategies for the land allocation problem. In general, these solution strategies can be divided into three categories. Some of the earliest solutions were based on linear programming (LP) (Dyer et al., 1983 and 1979; Johnson and Scheurman, 1977; Curtis 1962). LP-based models have the advantage of being relatively easy and fast to solve, but suffer from the disadvantage of producing solutions that are not directly mappable (i.e., they tend to produce solutions that indicate that X number of acres should be allocated to a particular land use, but they do not indicate precisely which X acres should be so allocated). A variant of this approach involves goal programming (GP) instead of LP, but for the purposes of this discussion, models based on GP generally have the same strengths and weakness as LP-based models.

A more recent alternative to LP- and GP-based land allocation models involves integer programming (IP)-based systems (Dean, 1996; Clements et al., 1990; Nelson et al., 1988). Unlike LP- or GP-based models, IP-based systems can produce solutions that are directly mappable. As a result of this mapability, IP-based models have been successfully linked to GIS (Dean, 1996). Unfortunately, IP models fall into the NP-complete class of computer problems, meaning that the amount of time and computer resources needed to solve IP models grows exponentially as the size of the problem increases. This means that solving even moderately large IP problems can be prohibitively computer intensive and impractically time consuming. Furthermore, IP-based systems generally depend upon an *a priori* partitioning of the study area into management units. This reduces the land allocation problem into a problem of assigning land uses to the preconceived management units. In general, IP-based models cannot deviate from this *a priori* partitioning; alternative partitioning that may produce superior solutions are not considered.

A number of hybrid models based on mixed integer programming (MIP) have been developed (Hof et al., 1994; Hof and Joyce, 1993). These models try to combine the strengths and minimize the weaknesses of the LP/GP and IP approaches. In general, none of these hybrid models has been entirely successful in achieving this goal. Furthermore, many of these MIP-based models are very situation-specific, and therefore lack the generalizability of the LP/GP- and IP-based approaches.

An alternative to all of these mathematical programming-based approaches is described by Eastman (1995). If the mathematical programming-based approaches can be said to emphasize the econometrics aspects of the land allocation problem, Eastman's approach emphasizes the geomatic side of the problem. Eastman's model is not entirely free of econometrics influences; at is heart it could be described as employing the economic concept of comparative advantage (CA). However, it is clearly a geomatic approach to the land allocation problem.

In a CA-based model, a raster **suitability map** is created for each possible land use. These suitability maps quantify the production potential of each raster cell relative to particular land uses. For example, consider a simplistic situation where the only two land uses being considered are timber production and agriculture. Now consider a particular raster cell within the study area where soils are moderately productive but somewhat rocky, slopes are moderate, and access is marginal. This cell would probably have a relatively low potential for producing agricultural outputs, and hence will be assigned a low value (say 1.5) in the agricultural production potential map. This implies that this cell is capable of producing 1.5 units of agricultural outputs.

This same cell might have a higher potential for producing timber products, because the cell's rocky soils and moderate slopes are not as much of a hindrance to timber operations as they are to agricultural activities. Thus, the cell would be assigned a relatively large value (say 2.5) in the timber production potential map. Again, this implies that the cell is capable of producing 2.5 units of timber outputs.

It is tempting to stop the analysis at this point and say that this hypothetical raster cell should be allocated to timber production, because the cell is clearly better suited to timber production than it is to agricultural production. From an economic viewpoint, this would be an **absolute advantage** analysis, where cells are assigned to land uses based on their ability to produce the outputs entailed by the competing land uses. However, an absolute advantage analysis of this sort fails to take into account the relative value of timber and agricultural outputs. Suppose that a unit of agricultural output is worth 4 units of value, while a unit of timber production is worth only 2 units of value. In this case, the agricultural output of the hypothetical cell is worth $1.5 \times 4 = 6$ units of value, while the timber output of the cell is worth only $2.5 \times 2 = 5$ units of value. Thus, the value of the outputs the cell is capable of producing indicate that the cell should be allocated to agricultural production. An analysis of this sort that assigns cell to land uses based on the value of the outputs cells are capable of producing is a **comparative advantage** analysis, and this is what forms the heart of Eastman (1995)'s approach[1].

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[1] Note that many land uses produce non-market outputs such as aesthetic qualities, wildlife habitat, and so on. The non-market nature of these outputs undeniably complicates the implementation of the process being described, but it in no way changes the basic theoretic concepts upon which the process is built.

The CA-based approach produces mappable outputs, is not dependent upon any *a priori* partitioning of the study area[2], and does not make intensive use of computer resources or require inordinate amounts of time to solve. Given these advantages, its seems to be at least the equal of the mathematical programming-based approaches. However, the CA approach is not without its flaws. Most significantly, the simple CA approach just described is a cell-by-cell analysis; there is no ability to account for spatial interactions between cells. In practice, this does not seem logical. For example, a raster cell that has been assigned to a land use involving wildlife habitat protection but is completely surrounded by raster cells assigned to a land use involving residential development would clearly be influenced by the nature of the surrounding cells, but the CA analysis just described would fail to take this interaction into account. IP-based models (and to a lesser extent, LP/GP-based models) have some ability to account for these sorts of spatial interactions through the use of adjacency constraints (Murry and Church, 1995; Yoshimoto and Brodie, 1994; Jones et al., 1991), but the simple CA-based model just described has no such capability.

This report will describe a new extension to the comparative advantage approach that is designed to give CA-based models the ability to account for spatial interactions. This approach utilizes both the classic CA model and a unique form of goal programming. In an ongoing study, this new approach is being compared to both the classic CA approach and various mathmatical programming-based approaches tp determine the strengths and weaknesses of the various solutions to the land allocation problem.

EXPANDING THE COMPARATIVE ADVANTAGE APPROACH TO INCLUDE SPATIAL INTERACTIONS

Consider a comparative advantage land allocation problem involving three land uses --A, B, and C. Further assume that the suitability maps and relative values for these uses are as shown on the left side of Figure 1. Given these suitability maps and relative values, the production value maps shown in the middle of Figure 1 would be produced, and the comparative advantage land allocation map shown on the right hand side would result.

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[2]Some might argue that dividing the study area into raster cells represents an *a priori* partitioning of the region. However, this partitioning is completely arbitrary and can be changed at will. Furthermore, as raster cells become smaller, the significance of this partitioning decreases.

Production Value Maps

CA-Based Land

Suitability

1.00	1.10	1.20	1.10
1.80	1.90	2.10	1.70
2.40	2.20	1.90	1.30

Relative 1.5

Production

1.50	1.65	1.80	1.65
2.70	2.85	3.15	2.55
3.60	3.30	2.85	1.95

Suitability

1.50	1.10	2.10	2.20
1.40	1.80	2.20	1.30
1.80	2.00	1.90	1.20

Relative 1.4

Production

2.10	1.54	2.94	3.08
1.96	2.52	3.08	1.82
2.52	2.80	2.66	1.68

CA Land

B	C	B	B
A	A	A	C
A	A	A	C

Suitability

1.40	1.50	1.90	2.10
1.60	1.60	2.00	2.20
1.70	1.90	2.00	2.20

Relative 1.3

Production

1.82	1.95	2.47	2.73
2.08	2.08	2.60	2.86
2.21	2.47	2.60	2.86

Figure 1. A schematic of the standard comparative advantage (CA)-based solution to the land allocation problem.

The final land allocations map from Figure 1 maximizes the total value of outputs produced from the mapped area, ignoring all interactions between cells. Now consider the "true" land allocation map for the same area shown in Figure 2. This map shows "actual" land uses in the hypothetical study area. If we assume that the true land allocation map reflects land uses decisions made by experts who took spatial interactions into account when land uses were selected, the difference between the CA-based land use map and the actual land use map should be due to the comparative advantage model's inability to consider spatial interactions.

CA Land Allocations

B	C	B	B
A	A	A	C
A	A	A	C

"True" Land Allocations

B	B	B	C
A	A	B	C
A	A	A	C

Figure 2. CA produced land allocations and a "true" land allocations map created by hypothetical experts.

Within the CA model, there are two ways of explaining spatial interaction effects. One is to assume that these effects influence the ability of cells to produce outputs (e.g., the fact that cell **X** is surrounded by cells allocated to a timber production land use influences the ability of cell **X** to produce output for a recreation land use). This would imply that a modification of the CA model designed to consider spatial interaction effects should in some way modify the values shown in the suitability maps. Alternatively, it is possible to view spatial interactions as influencing the relative values of the outputs produced by cells (e.g., the fact that cell **X** is surrounded by cells allocated to timber production influences the value of the recreation outputs produced by **X**). This implies that a modification to the CA model designed to consider spatial interactions should modify the relative values of the various land uses being evaluated.

In reality, it is probably possible to find examples of both types of spatial interaction effects, and it seems plausible to assume that in most situations both types of effects are acting simultaneously (e.g., adjacent land uses impact both the ability of a cell to produces outputs for land use **A** and the quality, and hence the value, of the unit **A** outputs produced by the cell). Fortunately, since production potentials and relative values are multiplied together in the CA model, the ultimate impact of either type of effect on the CA model's output will be identical (e.g., a 20% reduction in the production potential of cell **X** for some land use **A** has the same impact on the CA model as does a 20% reduction in the relative value of land use **A** for cell **X**). This implies that it should be possible to approximate the effects of both types of spatial interactions by modeling only one type. In this study, it will be assumed that all spatial interactions can be approximated by modifying relative values.

The classic CA model assumes that relative values are scalars that are applied uniformly across all cells in each suitability map. In the modified CA model developed here, relative values are cell-specific, so the relative value for land use **A** for cell **X** is likely different from the value of land use **A** for cell **Y**. In this approach, the relative value of each land use for each cell is a function of the land uses of adjacent cells. There are numerous ways to form this function, but for the purpose of this study, the relative value for a particular land use in a given call will be computed as follows:

$$RV_{AX} = \sum_{d=1}^{m} \left(\beta_{d0} D_d + \sum_{i=1}^{n} \beta_{di} PN_{D_d i} \right) \tag{1}$$

Where:

RV_{AX} = The relative value for land use A in cell X.

m = Number of neighborhoods being evaluated.

β_{xx} = Equation parameters.

D_d = Number of layers of cells around the cell of interest that make up the neighborhood being analyzed. A D_d value of 1 implies that the neighborhood extends one layer of cells out from the cell of interest and thus includes all eight immediately adjacent cells, a D_d value of 2 implies that the neighborhood extends two cell layers out from the cell of interest and thus includes the 24 closest cells, etc.

n = Number of land uses being evaluated.

$PN_{D_d i}$ = Percentage of cells in the neighborhood of size D_d around cell X that are allocated to land use i.

This equation makes certain assumptions regarding the nature of spatial interactions. Specifically, it assumes that the relative values of land uses are linearly related to the percentage of neighboring cells that are allocated to each of the various land uses being considered. The linearity assumption is somewhat arbitrary and used mainly for mathematical convenience. The assumption that the impacts of neighboring land uses can be measured through percentages of cells allocated to various land uses is again somewhat arbitrary, but it is at least a plausible way of measuring adjacent land uses. This percentage approach has the advantage of being quite simple. Thus, while it is certainly possible to envision other, more complex ways of measuring the impacts of neighboring land uses on relative values (e.g., fragmentation of the neighboring landscape, degree of connectivity of the current cell with other cells being used for identical and/or compatible uses, etc.), the percentage of neighboring cells approach serves as a viable starting point for this research.

Before equation (1) can be applied, the precise nature of the neighborhoods used in the equation must be defined. All neighborhoods are assumed to be symmetric, containing all the cells in a region centered on the cell of interest (except the cell of interest itself). Thus, the smallest neighborhood consists of the eight cells immediately adjacent to a given cell, the second smallest neighborhood consists of the 24 cells in the two "layers" of cells surrounding the cell of interest, and so on.

Neighborhood size is defined by the D_d term in equation (1). In effect, this term measures the number of "layers" of raster cells around the cell of interest that define the neighborhood. Thus, a D_d value of 1 indicates that the neighborhood is composed of the eight cells making up the first layer of cells around the cell of interest, a D_d value of 2 indicates that the neighborhood is composed of the 24 cells in the two layers of cells surrounding the cell of interest, and so on.

Equation (1) supports the simultaneous use of multiple neighborhoods of different sizes (the first summation operation in the equation aggregates across the m different neighborhoods being considered). This capability to consider multiple neighborhood sizes is included because it seems plausible that the effects of neighboring land uses might differ with differing neighborhood sizes (e.g., it is plausible that land use A might have a significant negative impact on land use B if the two uses are applied to raster cells that are immediately adjacent to one another, but the magnitude of this negative impact might decreases as the distance between the cells where the two land uses are applied increases). Note that while equation (1) includes the ability to consider multiple neighborhoods simultaneously, if m is set to 1 the first summation in the equation essentially dissolves and the equation reverts back to evaluating only a single neighborhood.

Using equation (1), the suitability maps from Figure 1, and the true land use map from Figure 2, it should be possible to derive values for the ? parameters of equation (1). Optimal ? values should reproduce (as accurately as possible) the true land uses. This implies that these ? parameters could then be applied to similar land use problems in areas where no true land use map exists. The result would be CA-based land use recommendations for this new area that explicitly consider the impacts of adjacent land uses.

Deriving ? values is not trivial. Simple regression-type analyses are not adequate, because there are n (the number of land uses being evaluated) different equations whose parameters must be estimated simultaneously. Seemingly unrelated regression (SUR) techniques could be used to overcome this problem, but then there is the issue of the non-symmetric error function that must be used to estimate ? values. This non-symmetry arises from the fact that the consequences of overestimating and underestimating relative values are not equal. For example, ? values that cause the relative value of the true land use for a given cell to be overestimated are not a problem, because an overestimation of this sort still leads the CA process to conclude that the true land use is the most desirable use for the cell in question. However, ? values that cause the relative value of the true land use for a given cell to be underestimated are a major problem, because underestimation will cause the CA process to conclude that the true land use is not as desirable as some alternative use for the cell in question. Similar logic can be used to determine that underestimation of the relative values of land uses other than the true land use for a given cell are not a problem, while overestimation of these values can be a problem. The result of this is the error function described in Table 1.

Table 1. Nature of the error function to be used to estimate ? values for equation (1).

	"True" Land Use	Other Land Uses
Overestimate Relative Value	Inconsequential	Problem to be avoided
Underestimate Relative Value	Problem to be avoided	Inconsequential

Standard regression techniques assume that the error function used to evaluate the goodness-of-fit of a particular set of ? values is symmetric. Since the error function described in Table 1 is not symmetric, regression techniques are inadequate for use in this situation.

Goal programming (GP) provides a viable method of estimating ? values for equation (1). GP can handle both the non-symmetric error function and the fact that parameters for multiple equations are being estimated. Formulating an appropriate goal programming problem is straightforward. Each cell generates n (the number of land uses being evaluated) constraints, one for each of the n equations being created. Assume that m (the number of neighborhoods to be evaluated) is 1 and only a neighbor of size 1 will be evaluated. For the upper left cell (the cell in row 0, column 0) and the next cell to the right (the cell in row 0, column 1) in our example from Figures 1 and 2, the constraints generated would be:

$$(1.0\times(((O_{DA}-U_{DA})\times 1)+((O_{\%AA}-U_{\%AA})\times 66.67)+((O_{\%BA}-U_{\%BA})\times 33.33) + ((O_{\%CA}-U_{\%CA})\times 0))-2.1)+(O_{00A}-U_{00A})=0 \quad (2)$$

$$(1.5\times(((O_{DB}-U_{DB})\times 1)+((O_{\%AB}-U_{\%AB})\times 66.67)+((O_{\%BB}-U_{\%BB})\times 33.33) + ((O_{\%CB}-U_{\%CB})\times 0))-2.1)+(O_{00B}-U_{00B})=0 \quad (3)$$

$$(1.4\times(((O_{DC}-U_{DC})\times 1)+((O_{\%AC}-U_{\%AC})\times 66.67)+((O_{\%BC}-U_{\%BC})\times 33.33) + ((O_{\%CC}-U_{\%CC})\times 0))-2.1)+(O_{00C}-U_{00C})=0 \quad (4)$$

$$(1.1\times(((O_{DA}-U_{DA})\times 1)+((O_{\%AA}-U_{\%AA})\times 40.00)+((O_{\%BA}-U_{\%BA})\times 60.00) + ((O_{\%CA}-U_{\%CA})\times 0))-1.54)+(O_{01A}-U_{01A})=0 \quad (5)$$

$$(1.1\times(((O_{DB}-U_{DB})\times 1)+((O_{\%AB}-U_{\%AB})\times 40.00)+((O_{\%BB}-U_{\%BB})\times 60.00) + ((O_{\%CB}-U_{\%CB})\times 0))-1.54)+(O_{01B}-U_{01B})=0 \quad (6)$$

$$(1.5 \times (((O_{DC} - U_{DC}) \times 1) + ((O_{\%AC} - U_{\%AC}) \times 40.00) + ((O_{\%BC} - U_{\%BC}) \times 60.00) + ((O_{\%CC} - U_{\%CC}) \times 0)) - 1.54) + (O_{01C} - U_{01C}) = 0 \quad (7)$$

Note that in all of these constraints, each ? value is represented by two GP variables (O_{xx} and U_{xx}). This is necessary in the GP environment because all GP variables are assumed to be non-negative. Thus, positive ? values are represented by the GP variables with positive coefficients (the O_{xx} variables in each equation) while negative ? values are represented by the GP variables with negative coefficients (the U_{xx} variables in each equation).

Each constraint multiplies a cell's suitability value for a given land use (using equation 2 as an example, this is the suitability value of the upper left cell for land use A, which is 1.0) with the "new" relative value of that land use as computed using equation (1). This new relative value is computed using percentage of neighboring cells figures obtained from the true land use map shown in Figure 2. The result of this simple multiplication operation is the new production value for the cell and land use in question. The "old" production value of the true land use for the cell in question (as computed using the old relative value; in the case of equation 2, this old relative value is 1.5 and the old production value is 2.1) is then subtracted from the new production value just computed. The objective of the GP problem will be to ensure that constraints describing true land uses have net positive values (thereby ensuring that the new production values of these cells/land use combinations exceed the old production values for the cells' true land uses) while simultaneously ensuring that constraints describing other land uses have net negative values (thereby ensuring that the production values of these cell/land use combinations are less than the old production values for the cells' true land uses). If this objective can be fully meet, the final result will be a set of new production values that produce a CA output map that exactly matches the true land use map. Of course, it may not be possible to fully meet this stated objective, but the GP process will ensure that the objective is meet as fully as possible.

The last two GP variables in each constraint provide the necessary link to the objective function. In equation (2), these are the O_{00A} and U_{00A} variables. Since each constraint is forced to sum to zero, O_{00A} will take on a nonzero value if the remainder of the constraint has a net negative value (indicating that the cell's new production value is less than the old production value for the cell's true land use), and U_{00A} will take on a nonzero value if the remainder of the constraint has a net positive value (indicating that the cell's new production value is exceeds the old production value for the cell's true land use). With these variables in hand, constructing the GP model's objective function (and thereby completing the model) is simply a case of picking (for each constraint) one of these final variables to minimize. In our example problem, the final objective function would be:

$$\begin{aligned} Minimize = {} & U_{00A} + O_{00B} + U_{00C} + U_{01A} + O_{01B} + U_{01C} + U_{02A} + O_{02B} + U_{02C} + \\ & U_{03A} + U_{03B} + O_{03C} + O_{10A} + U_{10B} + U_{10C} + O_{11A} + U_{11B} + U_{11C} + \\ & U_{12A} + O_{12B} + U_{12C} + U_{13A} + U_{13B} + O_{13C} + O_{20A} + U_{20B} + U_{20C} + \\ & O_{21A} + U_{21B} + U_{21C} + O_{22A} + U_{22B} + U_{22C} + U_{23A} + U_{23B} + O_{23C} \end{aligned} \quad (8)$$

The first three terms in this objective function relate to the upper left cell in the map (i.e, the cell in cell in row 0, column 0, hence the subscripts "00" defining this cell). The first term relates to land use A for this upper leftmost cell, and causes the GP model to minimize the amount by which the first constraint (equation 2) is less than zero. This has the effect of causing the new relative value created by the constraint to produce a production value less than the old production value for the true land use of the cell (in this case, the true land use for the upper leftmost cell is use B, and the old production value for use B is the 2.1 that appears in all three of the constraints relating to this cell). The second term relates to land use B in the upper leftmost cell and causes the GP model to minimize the amount by which the second constraint (equation 3) is greater than zero. This has the effect of causing the new relative value generated by the constraint to produce a production value that exceeds the old production value for the cell's true land use. Taken together, these two constraints and their associated objective function values cause the GP model to create relative values that will cause the true land use (use B) to have the a higher production value than the alternative land use (use A). When this logic is applied to all of the constraints (which describe each land use in each cell), one can see why this GP formulation will find equations to compute relative values that minimize deviations from the true land use map.

When the GP model just described is formulated and solved for our example problem (the final GP problem had 36 constraints and 96 variables, and was solved in 16 steps in less than 1/16 of a second of CPU time), the following relative value computation formulas were produced:

$$RV_A = 0 \times D_d + 0.0205 \times \%NC_A + 0.0097 \times \%NC_B + 0.0121 \times \%NC_C \quad \textbf{(9)}$$

$$RV_B = 0 \times D_d + 0.0176 \times \%NC_A + 0.0116 \times \%NC_B + 0.0141 \times \%NC_C \quad \textbf{(10)}$$

$$RV_C = 0 \times D_d + 0.0103 \times \%NC_A + 0.0103 \times \%NC_B + 0.0185 \times \%NC_C \quad \textbf{(11)}$$

Where:

RV_X = The relative value of land use X in the current cell.

D_d = The neighborhood size (always 1, in this example).

NC_X = The percentage of cell in the neighborhood allocated to land use X.

With these equations in hand, it is possible to create a modified CA approach that explicitly considers spatial interactions. This modified CA approach can be applied to any similar land allocation problem where the interactions between adjacent land uses can be assumed to be identical (or nearly so) to the interactions used to generate the relative value equations.

The modified CA model simply replaces the map-wide relative values shown in Figure 1 with new relative value maps, where each cell has a unique relative value computed using the equations obtained from the GP analysis. The only difficulty in this procedure is determining what percentages of neighborhood cells have been allocated to various land uses (because there is presumably no "true" land use map from which to extract these values), but even this difficulty is easily overcome. All that is required is to perform the analysis using any arbitrary percentages of neighborhood cells, construct a land use map based on these arbitrary values, and calculate new percentages based on the resulting land use map. Re-perform the analysis using these new percentages and note any land use changes between the new land use map and the land use map from the previous iteration. Repeat the process of constructing new land use maps and re-computing neighborhood percentages until the system converges upon a land use map that does not appreciably change from one iteration to the next.

As an example of this procedure, the relative value equations (equations 9 through 11) were applied to the suitability maps from Figure 1 to produce the new land use map shown in Figure 3. The process was started by assuming that each cell had an equal percentage of neighboring cells in all possible land uses (i.e., each cell's neighbors were divided equally among land uses A, B and C). The process converged on the solution shown in Figure 3 in 18 iterations.

CA Allocations without Spatial Interactions

B	C	B	B
A	A	A	C
A	A	A	C

"True" Land Allocations

B	B	B	C
A	A	B	C
A	A	A	C

CA Allocations with Spatial Interactions

B	C	B	C
A	A	B	C
A	A	A	C

Figure 3. CA produced land allocations with no spatial interaction, "true" land allocations, and CA-produced land allocations constructed considering spatial interaction.

CONCLUSIONS

The modified comparative advantage model described here offers the promise of being able to combine most of the strengths and avoid most of the weaknesses of the various existing solutions to the land allocation problem. Like the classic LP/GP based solutions, the new approach is very fast and efficient, and the difficulty of finding a solution does not grow exponentially as problem size increases. Furthermore, like the IP-based land allocation solutions, the new approach is explicitly spatial in nature, but unlike the IP model, the modified CA process does not require any *a priori* division of the analysis area into management units. Unlike the classic CA-based approach, the technique proposed here has the ability to consider spatial interactions between neighboring land uses. Figure 3 makes the impact of this ability clear. The classic CA approach produces a result that differs from the hypothetical true land allocation by three cells, while the modified approach differs from the true solution by only one cell.

The new technique is not without its own unique weaknesses. Most notably, it cannot create a solution for a new land allocation problem until it can be calibrated using a known solution for an related land allocation problem. In addition, the new technique has yet to be tested and validated. Further study are underway to perform these tasks.

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